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Suggested Plume Rise Algorithm for Small Point Sources

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The use of coal is increasing as a source of fuel for small, industrial boilers with exhaust gases vented to relatively short stacks. Conversions by gas or oil users to coal fired boilers, increase the amount of pollutants that are emitted to the air and enhance the importance of adequately predicting the ground level pollutant concentration due to the source. Accounting for the rise of the plume and the presence of stack aerodynamic effects on the plume has become critical. This paper discusses the history of the development of analytical techniques for determining the rise of a plume after its release, particularly when the physical stack structure inhibits the rise of the plume. Also, an algorithm is presented that represents state-of-the-art methodologies for determining the plume rise. The methodology uses parameters which are generally available to the air quality scientist.

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Division of Air Pollution Control
Illinois Environmental Protection Agency

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Air Quality Planning Section

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Abstract

The use of coal is increasing as a source of fuel for small, industrial boilers with exhaust gases vented to relatively short stacks. Conversions by gas or oil users to coal fired boilers, increase the amount of pollutants that are emitted to the air and enhance the importance of adequately predicting the ground level pollutant concentration due to the source. Accounting for the rise of the plume and the presence of stack aerodynamic effects on the plume has become critical. This paper discusses the history of the development of analytical techniques for determining the rise of a plume after its release, particularly when the physical stack structure inhibits the rise of the plume. Also, an algorithm is presented that represents state-of-the-art methodologies for determining the plume rise. The methodology uses parameters which are generally available to the air quality scientist.

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Abbreviations and Symbols

C_p	- specific heat capacity of air at constant pressure (cal/gm-deg)
D	- inside stack diameter (m)
F	- buoyancy flux parameter (m^4/sec^{-3})
Fr	- Froude Number = $w(g(\Delta\rho/\rho_a)D)^{-1/2}$ (dimensionless)
g	- acceleration due to gravity (9.8 m/sec^2)
H	- production rate of turbulence due to convection = $g/(C_p\rho_a T_a)$ times the sensible heat flux at the surface (cm^2/sec^3)
h,	- height above the ground, (m)
h'	- negative stack downwash increment (m)
h_e	- effective stack height after stack tip downwash = $h_s + h'$ (m)
h_s	- stack height (m)
Δh	- plume rise (m)
Δh_f	- final plume rise (m)
k	- von Karman constant = 0.35 to 0.40 (dimensionless)
L''	- Monin-Obukov scaling length = $-u^{*3}/(kH)$ (m)
R	- velocity ratio =w/u (dimensionless)
R_{crit}	- cross-over value of R between different stack downwash regions in Figure 1 (see also equations 8 and 9) (dimensionless)
T_a	- ambient atmosphere air temperature (°K)
T_s	- stack efflux temperature (°K)
ΔT	- temperature difference between the stack efflux gas and the atmosphere = $T_s - T_a$ (°K)
u	- wind speed at stack height (m/sec)
u^*	- friction velocity (m/sec)
V_f	- stack gas volume flow (m^3/sec)
w,	- stack efflux velocity (m^3/sec)
w'	- mean vertical velocity of vertical plume (m/sec)
x	- downwind distance from source (m)
x_f	- distance downwind to final rise (m)
x^*	- downwind distance at which atmospheric turbulence begins to dominate entrainment (m)
z	- height of plume centerline above the stack top (m)
z_o	- roughness length of surface (m)
β	- entrainment coefficient for a bent-over plume (dimensionless)
n	- coefficient relating plume dissipation rate to w'^{-3}/z (dimensionless)
ρ_a	- density of the atmosphere (g/m^3)
ρ_s	- density of the stack gases (g/m^3)
$\Delta\rho$	- density difference between the efflux stack gases and the atmosphere = $\rho_a - \rho_s$ (g/m^3)

Introduction and Statement of the Problem

As the price differential between oil and coal increases and the coming years see decontrol of natural gas prices, boilers that formerly burned natural gas and oil will most likely be converted to coal. Coal conversions can save money and allow precious gas and oil to be used for other purposes. However, coal is a dirtier fuel than either gas or oil and can pose severe air pollution problems, especially in terms of the major pollutants associated with coal -- sulfur dioxide, suspended particulates, and trace elements. Many of the anticipated fuel conversions will involve small industrial and institutional boilers, which are located in highly populated areas. Such sources may also have short stacks with low stack exit gas velocities and, therefore, be subject to stack or building aerodynamic effects (i.e., downwash).

The so-called "2/3's Law" (Briggs, 1969) has been successfully used in the past to develop approximations of stack plume rise from large sources. However, small point sources often come under the influence of certain phenomena which the "2/3's Law" is not capable of handling successfully. These phenomena have the potential to decrease plume rise and subsequently increase ground level air pollution impacts.

The most important of these phenomena are stack tip downwash, building downwash, and mechanical turbulence induced by wind sheer near ground level. The purpose of this paper is to outline a procedure which provides a reasonable approximation of plume rise from small sources which may undergo stack tip downwash and limited plume rise due to mechanical turbulence. The issue of building downwash will not be addressed in this paper and the reader is referred to Huber and Snyder (1976) and Huber (1977) for further details on the subject. This paper first examines the relevant literature discussing plume rise and secondly proposes a complete plume rise algorithm which reflects state-of-the art methodologies for predicting plume rise for neutral conditions and yields realistic, yet conservative results. No attempt will be made to provide rigorous derivations of the equations suggested, as these can be found in the literature. It must be stressed that the algorithm suggested in the paper represents the authors' concept of a workable system. We do not claim that it represents a theoretically "perfect" equation. The authors have made some necessary assumptions to obtain a set of equations that are operable with the input data which is normally available to air quality scientists. We fully realize that equations which are more precise could be derived if a controlled environment were available and if all unknowns could be measured precisely.

Literature Review

At this point, a brief discussion of the most relevant literature dealing with stack tip downwash and plume rise, especially as it applies to small point sources, is appropriate.

1. Sherlock and Stalker (1941) provides perhaps the first definitive assessment of stack downwash. The Sherlock and Stalker data was derived from a series of tests with a single isolated stack in a wind tunnel without augmentation of air stream turbulence. The tests were run at various combinations of wind speed, stack velocity and stack temperature. The heights of the lower plume boundary were measured and plotted versus downwind distance. The authors concluded that the tests showed that the "temperature of the gas is relatively unimportant as a factor in downward relationships." It should be noted that the tests were run without Froude number scaling (i.e., the Froude numbers were very high); therefore, the buoyancy effect was diminished, probably to a point where it was of negligible importance. Halitsky (1978) states that it is reasonable to expect that proper scaling of the Froude Number would have reduced downwash at the higher temperatures due to buoyancy.

   Sherlock and Stalker presented evidence which demonstrated that for a momentum dominated plume at a ratio of stack velocity (w) to wind speed (u) of 1.5 or greater (R = w/u), noticeable stack downwash should not occur. This is the so-called "1.5 Rule" for determining the presence of stack downwash.

2. Gifford (1972) suggests that the negative height increment, h', given in Equation (1), should be added algebraically to the physical stack height to obtain the effective plume centerline height after mixing has occurred in a stack wake.

$$h' = A(w/u - 1.5)D, \qquad (1)$$

   where w is the stack emission velocity, u is the wind speed at stack height, D is the inside stack diameter and A is a numerical constant equal to 3.0.

3. Briggs (1973, 1976) suggests that the constant A in Equation (1) should be 2.0. The data base for Equation (1) is described in Briggs (1973, 1976) as the wind tunnel observations of Sherlock and Stalker (1941). These observations showed the following: 1) downwash (negative rise) occurs when v_s/u is less than 1.5 and 2) the bottom of the plume downwashes about one stack diameter at $v_s/u = 1$. Briggs notation v_s/u is synonymous with w/u in Equation (1). Equation (1), as applied to the centerline by Briggs (1973, 1976) with A equal to 2.0, is more conservative than the Sherlock and Stalker (1941) equation since they addressed the lower bound of the plume, not the centerline. However, at values of R which are less than 1.0, Halitsky (1978) showed that Briggs (1973, 1976) provides a reasonable estimate of the height of plume centerline.

4. Briggs (1969, 1970) used dimensional analysis of three conservation equations along with limited empirical data for closure techniques to develop a series of equations that applied to buoyant rise in neutral and unstable atmospheric conditions. This is the so-called "2/3's Law" for buoyant rise.

$$\Delta h = 1.6F^{1/3} u^{-1} x^{2/3} \quad (2)$$

Briggs (1970) estimated the distance to final rise (x_f) to be

$$x_f = 3.5\, x^*, \quad (3)$$

where x^* is the distance downwind at which atmospheric turbulence begins to dominate over buoyancy. Final rise is then equal to the equation:

$$\Delta h_f = 1.6F^{1/3}u^{-1}(3.5\, x^*)^{2/3} . \quad (4)$$

The variable Δh_f may be considered as the effective final plume height, since during neutral conditions actual plume rise may never terminate (at least until the plume reaches the top of the mixing layer).

5. Briggs (1975, 1982) revised the final plume rise formula to account for rise which is limited by mechanical turbulence. Mechanical turbulence (due to wind shear) dominates over convectively produced turbulence as long as the following is satisfied:

$$h_s < |L''| , \quad (5)$$

where h_s is the physical stack height, $L'' = -u^{*3}/(kH)$ is the Monin-Obukhov scaling length, u^* is the friction velocity, k is the Von Karman constant (measurements of k range from 0.35 over very smooth surfaces to about 0.40 over more typical ground cover) and H is the production rate of turbulence due to convection.

The final plume rise for conditions where mechanical turbulence dominates convective turbulence is given by Briggs (1975) as follows:

$$\Delta h = 1.3(F/uu^{*2})(1 + h_s/\Delta h)^{2/3}. \quad (6)$$

For most sources, when mechanical turbulence dominates and atmospheric stability is neutral, $\Delta h < 2h_s$. Equation (6) can then be approximated by the following (Briggs 1975, 1982) equation:

$$\Delta h = 1.54(F/uu^{*2})^{2/3}h_s^{1/3}. \quad (7)$$

This is the so-called "Break-up" Model.

6. When Overcamp and Hoult (1971) studied plume downwash in the wake of a cooling tower, they found that the efflux Froude number (Fr) and the velocity ratio (R = w/u) were critical to whether the plume was affected by the tower aerodynamic wake. They investigated the effect of low Froude Numbers (i.e., high plume buoyancy) on the plume rise of the effluents emitted from a hyperbolic natural draft cooling tower. Overcamp and Hoult found that if the wake of the tower grows faster than the plume rises, then stack (tower) downwash occurs.

Using the results of the experiments by Overcamp and Hoult (and modifying their definition of the Froude Number to be consistent with its definition in this paper) the following equations can be derived:

(1) To identify the cross-over value of R (R_{crit}) between no interaction and slight interaction of the plume and the tower,

$$R_{crit} = 0.69Fr^{0.47} . \quad (8)$$

(2) To identify the cross-over value of R (R_{crit}) between slight interaction and strong interaction of the plume and the tower,

$$R_{crit} = 0.50\ Fr^{0.65} . \quad (9)$$

For values of R which are much less than R_{crit} in equation (9), the plumes strongly interact with the tower wake and the plume centerlines do not experience rise (or the rise takes place so far downwind that the maximum ground level impact occurs during the period when the plume rise is zero). For values of R much greater than R_{crit} in Equation (8), interaction is very weak or nonexistent. For R values greater than R_{crit} in Equation (9), but less than those values in Equation (8), slight interaction of the plume and the tower wake should be expected.

7. Huber, et. al. (1980) indicate that the downwash without the subsequent plume rise indicated by Overcamp and Hoult (1971) also occurs to some extent in the case of tall, thin stacks.

Discussion of the Relevant Literature

1. "Stack Tip Downwash"

Wind moving past a smokestack causes a low pressure area on the immediate lee side of the stack. If the effluent smoke plume lacks the ability to rise due to insufficient momentum or insufficient buoyancy, it may be drawn into the low pressure area in the wake of the stack and displaced downward. This is a stack tip aerodynamic effect, commonly referred to as stack tip downwash.

Traditionally, stack tip downwash calculations have been performed following Briggs (1973). Recommended USEPA computer models use this method for diffusion estimates (Pierce and Turner, 1980). As discussed earlier in this paper, Briggs' (1973) method subtracts an increment from the physical stack height and assumes that plume rise will otherwise be unaffected. Briggs (1981) stated that his work in 1973 represented an oversimplification of the pnenomena. Overcamp and Hoult (1971) and Huber, et al (1980) along with simple plume observations support this conclusion. Plume observations have indicated that, when a plume is sufficiently drawn into the stack wake, virtually no plume rise will occur at least

within the context of determining maximum ground level concentrations. The authors have attempted to consolidate these observations with the earlier plume rise computation techniques and believe the procedure which we discuss will realistically (yet conservatively) account for stack tip downwash.

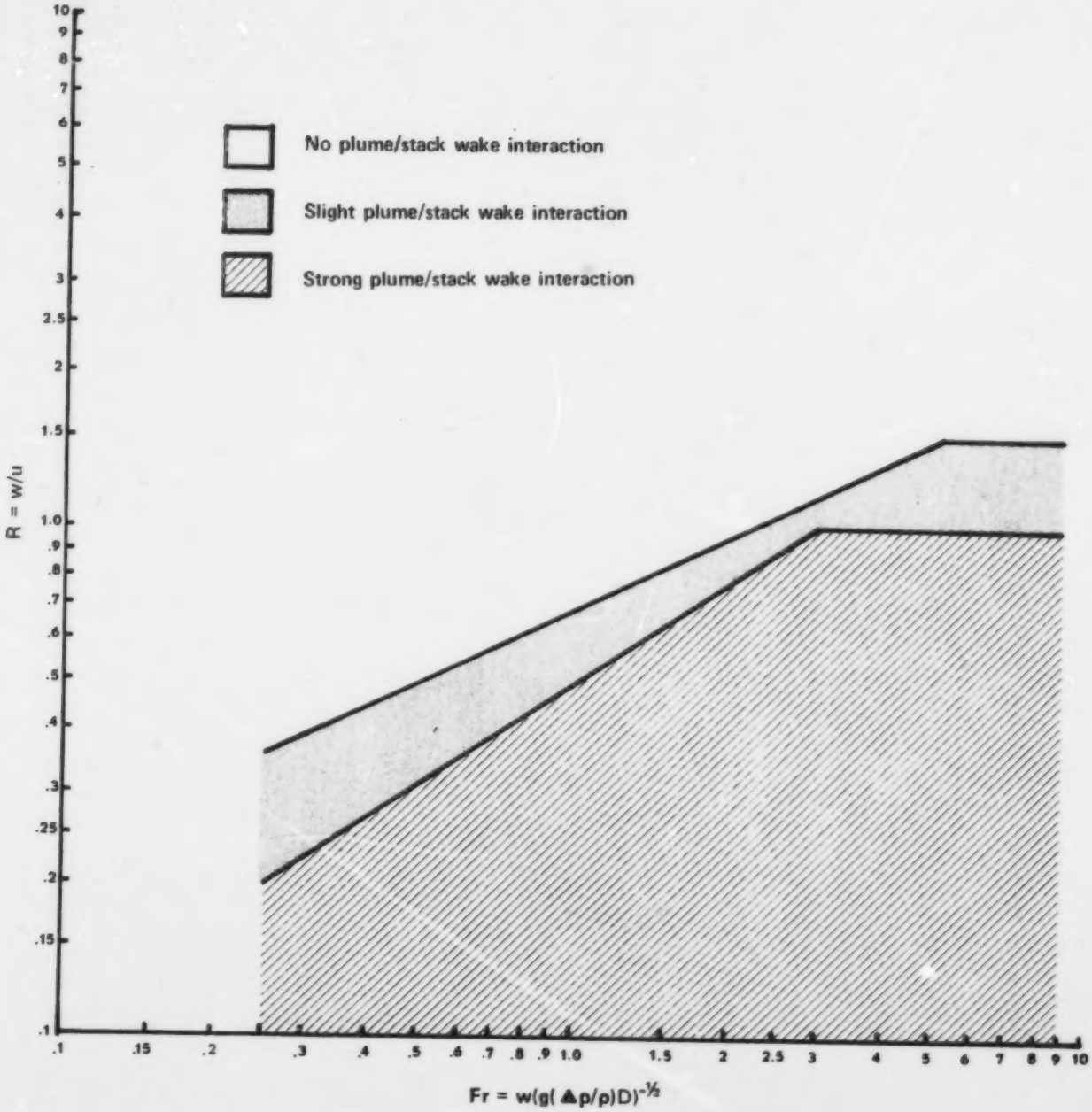
The data from the experiments of Overcamp and Hoult (1971), when rearranged to reflect no interaction, slight interaction and strong interaction of the plume and the stack wake, may be explained with the use of Equations (8) and (9). These equations indicate that for a very buoyant plume (i.e., if the efflux Froude Number (Fr) is small) the buoyancy forces are sufficiently large to counteract some of the adverse pressure forces causing the stack effluent to downwash (Briggs, 1969). Figure 1 graphically depicts Equations (8) and (9). For values which fall in the strong interaction (cross-hatched) area in the figure, the plume centerline is expected to drop below the stack tip and the plume will not experience rise. Equation (1), consistent with Briggs (1973), is adequate to calculate the height of the plume centerline for diffusion estimates. For values which fall in the slight interaction area of Figure 1, the plume is expected to initially dip somewhat before continuing to experience plume rise. Equation (1), consistent with Briggs (1973), provides an acceptable method of calculating plume rise during this slight plume/stack wake interaction. Since the plume does continue to rise after the interaction in the wake occurs, appropriate plume rise equations should also be applied. For values which are in the "no plume/stack wake interaction area", stack tip downwash is not expected and only the standard plume rise equations should be applied.

At large values of Fr, the plume rise approaches that of a pure momentum plume at distances very near the stack. This condition was examined by Sherlock and Stalker (1941). They found that slight downwash occurs if $R < 1.5$ and substantial downwash occurs if $R < 1.0$. Therefore, the far right-hand portion of Figure 1 appoaches the traditional "1.5 Rule" which is generally used by air quality scientists for predicting the occurrence of stack downwash. It is reasonable to assume that if the plume has sufficient momentum (i.e., $R > 1.5$), no matter what the degree of buoyancy in the effluent, the plume will not exhibit stack tip downwash. In other words, additional buoyancy can only help to further alleviate stack tip downwash.

The values of R_{crit} dividing the strong and the slight interaction areas of Figure 1 reach a maximum of $R = 1$ for large Froude Numbers. This value was selected by the authors as being generally consistent with "cut-off" values presented in the literature. Since the bottom of a momentum plume "sinks" about one diameter below the stack top at $R = 1$, it seems reasonable (pending data which would indicate a better location for this maximum) that plume centerlines would therefore "sink" less than one diameter and undergo slight plume/wake interaction. Until data for determining a more precise value for this "cut-off" becomes available, the value of $R=1$ provides a conservative estimate.

Figure 1

Stack Downwash as a Function of the
Froude Number and Velocity Ratio



2. "Rise Limited by Mechanical Turbulence"

The "2/3's Law" for plume rise during neutral and unstable atmospheric conditions provides an excellent approximation of plume rise from tall stacks, such as those associated with large fossil fueled utility plants. The plume rise (Δh) for the "2/3's Law" as stated in Equation (2) is as follows:

$$\Delta h = 1.6\ F^{1/3} u^{-1} x^{2/3}, \qquad (2)$$

where F is the buoyancy flux, u is the windspeed at stack tip, and x is the distance downwind. This equation assumes that the rate of plume rise is limited by the mixing of ambient air into the plume caused by the plume's own turbulence and the velocity shear between the stack gases and the air. In addition, at some point downwind the convective turbulence of a well-mixed atmosphere begins to dominate the general plume entrainment. At some distance still further downwind the plume effectively stops rising due to a decay in buoyancy. For small industrial sources with short stacks (often in urban souroundings), mechanical turbulence (due to wind shear and surface roughness elements) may be dominant over convective turbulence. Mechanical turbulence dominates over convective turbulence as long as

$$h_s < |L''| . \qquad (5)$$

Practically, however, this equation is difficult to evaluate since L", the Monin-Obukhov scaling length, is difficult to calculate with meteorological data which are generally available.

The most common descriptor of the scale of mechanically-produced turbulence near the surface is the friction velocity, u*. Direct measurement of u* is difficult. However, under neutral conditions u* may be estimated using the established logarithmic law (Briggs, 1975)

$$u = (u^*/k) \ln(h/z_0), \qquad (10)$$

where k is the von Karman constant, h is the height above ground, z_0 is the roughness length associated with the surface over which the air is moving, and u is the wind speed. As noted previously, the von Karman constant has been found experimentally to be about 0.35 over smooth surfaces and about 0.40 over typical ground cover.

The roughness length z_0 is about 1/10 the height of the surface elements such as trees, houses, grasses, crops, etc. It may range from 1 cm or less for short grasses to over 1 meter for a city. The following table summarizes z_0 values for various surfaces. Values for grasses decrease with increasing wind speed because the grasses bend in the wind.

Values of the Surface Roughness, z_o, for Various Surface Configurations (Strom, 1976)

Surface Configurations	z_o (cm)
Very smooth-mud flats and ice	0.001
Even snow cover of great depth	0.05
Snow surface-natural prairie	0.10
Desert (Pakistan)	0.03
Semi-desert	0.03
Bare solid ground	1.0
Ploughed land	2.0
Flat open country	3.0
Mown grass, h=1.4-4.5 cm	0.2 - 2.4
Various long grasses - thin and thick, h = 50-70 cm	3.7 - 9.0
Woodland forest	30
Town	100
City	200
Urban area	300
City (Liverpool)	123
City (Philadelphia-varying terrain)	22-310
City (Central Copenhagen)	750

Equation 10 is valid only for a neutral stability surface layer, which is limited in height by "L". Calculation of u* in non-neutral conditions requires the input of surface heat transfer data, which is usually not available for routine diffusion analyses.

The final plume rise for conditions where mechanical turbulence dominates convective turbulence is given by Briggs (1975) in his "Break-Up" model as follows:

$$\Delta h = (2/3\beta F/uu^{*2})^{3/5}(\eta k h_s)^{2/5} \quad (11)$$
$$= 1.3\,(F/uu^{*2})(1 + h_s/\Delta h)^{2/3} \quad (6)$$

where β is the entrainment coefficient which equals 0.6; η is the coefficient relating plume dissipation rate to w'^3/z and is equal to 1.5; k is the von Karman constant where 0.4 has been assumed.

When mechanical turbulence dominates, for most sources $\Delta h < 2h_s$ and Equation (12) can be approximated more simply by the following equation:

$$\Delta h = 1.54(F/uu^{*2})^{2/3}h_s^{1/3} \quad (7)$$

This is satisfactory if h_s exceeds $4F^{1/2}$ in mks units (Briggs, 1975).

Figure 2 presents a comparison between the "2/3's Law" (Equation (10)) and Briggs (1975) equation for plume rise limited by mechanical turbulence (Equation (6)). The curves represent the predicted plume rise

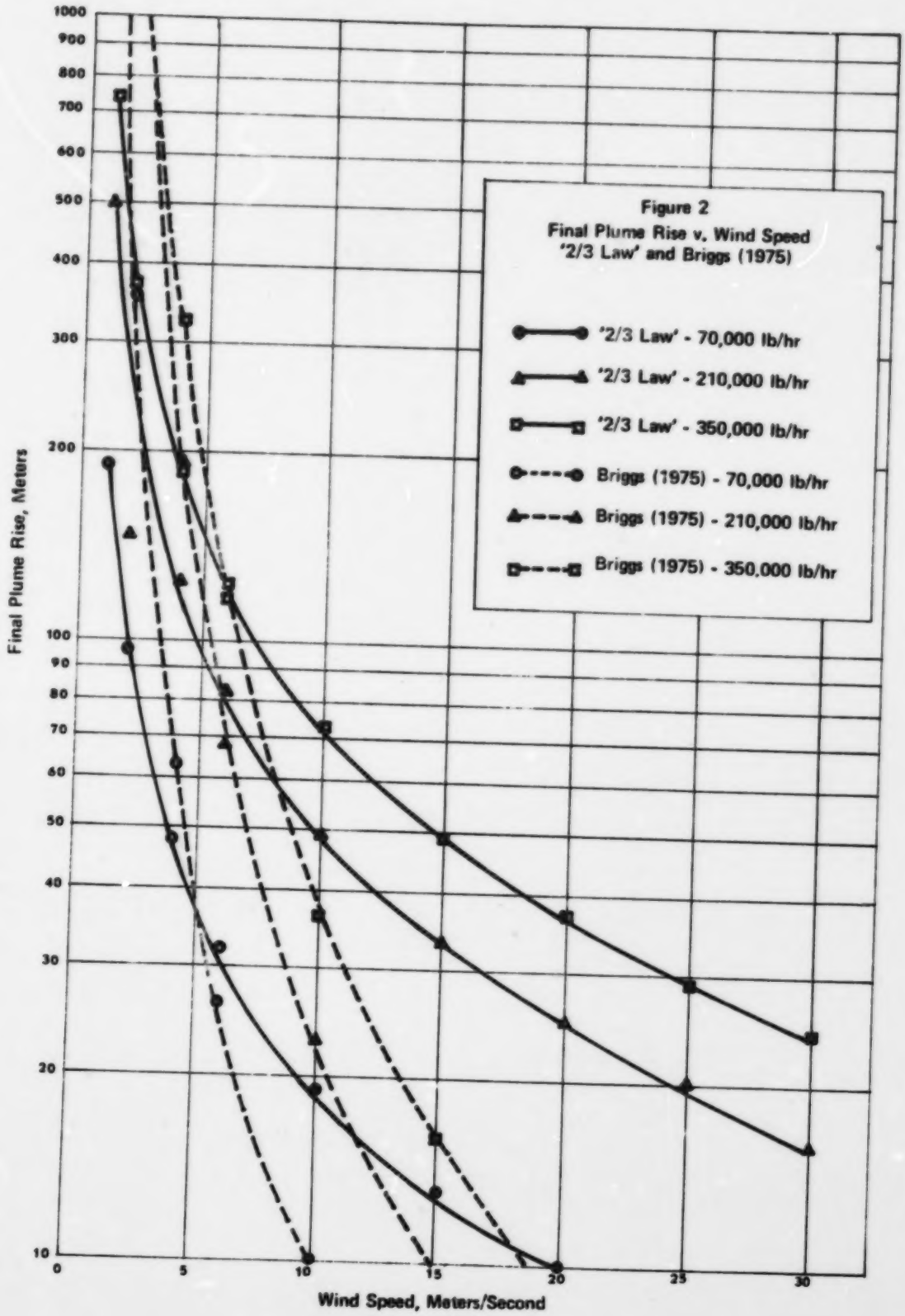


Figure 2
Final Plume Rise v. Wind Speed
'2/3 Law' and Briggs (1975)
'2/3 Law' - 70,000 lb/hr
'2/3 Law' - 210,000 lb/hr
'2/3 Law' - 350,000 lb/hr
Briggs (1975) - 70,000 lb/hr
Briggs (1975) - 210,000 lb/hr
Briggs (1975) - 350,000 lb/hr
Final Plume Rise, Meters
1000
900
800
700
600
500
400
300
200
100
90
80
70
60
50
40
30
20
10
0
5
10
15
20
25
30
Wind Speed, Meters/Second

of a typical industrial/institutional boiler with a stack height of 200 feet. This figure shows data for an operation at three different boiler loads plotted against wind speed. The assumed ambient temperature was 295°K and the stack exit temperature varied from a low of 425°K to a high of 490°K. The figures show that characteristically Briggs (1975) predicts lower plume rise values at high wind speeds than does the "2/3's Law".

Because it is difficult to calculate the Monin-Obukhov scaling length with meteorological data generally available to the air quality modeler, it is difficult to decide which plume rise formula to use (i.e., the "2/3's Law" or the Briggs (1975) so-called "breakup formula"). To provide a conservative estimate of plume rise, it is suggested that both formulas be evaluated. The one which yields the smallest Δh should be used for calculating plume rise for subsequent dispersion estimates.

A Suggested Plume Rise Algorithm for Small Low Level Point Sources

The following procedure provides an estimation of plume rise from small industrial sources with short stacks, which may be subject either to stack tip downwash or to a decrease in plume rise due to mechanical turbulence. For determining plume rise during stable conditions, the reader is referred to Briggs (1969, 1972). These procedures may also be used for estimating plume rise during unstable conditions, since during light wind conditions the procedure will approach the "2/3's Law".

Step 1. Calculate the Froude Number, Fr.

$$Fr = w(g(\Delta\rho/\rho_a)D)^{-1/2} \qquad (12)$$

For most effluent stack gases where the molecular weight is similar to air, the Froude Number simplifies to:

$$Fr = w(g(\Delta T/T_s)D)^{-1/2}, \qquad (13)$$

where ΔT is the difference between the ambient temperature and the temperature of the stack exit gas, T_s.

Step 2. Calculate the friction velocity, u*.

$$u^* = (uk)\ln(h/z_o) \qquad (14)$$

Step 3. Calculate the Buoyancy Flux parameter, F.

$$F = 3.12\ V_f(\Delta T/T_s) \qquad (15)$$

Step 4. Determine the stack downwash potential of the effluent plume. This is done by locating the Froude Number/Velocity Ratio (Fr/R) pair on Figure 1. If the Fr/R pair falls in the "no plume/stack wake interaction area" of the figure, go to Step 5. If the Fr/R pair falls in the "slight plume/stack wake interaction area" of the figure go to Step 6. If the Fr/R pair falls in the "strong plume/stack wake interaction area" of the figure, go to Step 7.

Step 5. Calculate the plume rise using the "2/3's Law".

$$\Delta h = 1.6F^{1/3}\, u^{-1}\, (x_f)^{2/3}, \qquad (2)$$

where:

x_f = the distance to final plume rise = $3.5x^*$,
$x^* = 14F^{5/8}$ for F less than 55,
$x^* = 34F^{2/5}$ for F greater than or equal to 55.

Calculate the plume rise using the "Break-Up" model using. either equation (6) or (7)

$$\Delta h = 1.3\, (F/uu^{*2})(1 + h_s/\Delta h)^{2/3} \qquad (6)$$

or

$$\Delta h = 1.54\, (F/uu^{*2})^{2/3} h_s^{1/3} \qquad (7)$$

Under most conditions, Equation (7) yields results similar to Equation (6) and it has the added benefit of being easier to calculate since its solution does not require an initial estimate of Δh.

Use the lesser of the two calculated plume rise values.

Step 6. Determine the plume release height by adding the following negative increment to the stack height, h_s:

$$h' = 2.0\, (R-1.5)D. \qquad (18)$$

The stack height used in dispersion calculations is then equal to the physical stack height h_s plus h', the negative stack downwash increment, or:

$$h_e = h_s + h' \qquad (19)$$

Next, use the procedure outlined in Step 5 to determine plume rise.

Step 7. Determine the plume release height using the technique described in Step 6. For dispersion estimates set plume rise, Δh, equal to zero.

Appendix A contains a copy of a computer subroutine which can be used for calculating plume rise from small point sources. The subroutine applies the suggestions in this paper to the standard USEPA plume rise subroutine BEH072. The subroutine can be incorporated in the appropriate dispersion model for use in determining plume rise from small point sources using the procedure outlined above.

Conclusion

The procedure outlined above reflects a workable technique for determining plume rise from small industrial sources with short stacks. It should enable the dispersion modeler to approximate the impacts of small sources on air quality more accurately than through the sole use of the "2/3's Law".

This procedure uses several assumptions that will most likely lead to slightly conservative estimates. Slight conservatism is often desirable in dispersion modeling for comparison with air quality standards. The conservatism is due in large part to the following:

Since the Overcamp and Hoult (1971) data was collected for large hyperbolic-shaped natural draft cooling towers which have large cross-sectional areas, equations (8) and (9) probably reflect a substantial amount of building wake affect in addition to stack downwash.

Fay, et al (1970) found that the "1.5 Rule" may be conservative. For plumes with both momentum and buoyancy, a more appropriate rule may be a "1.2 Rule".

References

Briggs, G.A., (1969): Plume Rise, USAEF Critical Review Series, TID-25075.

Briggs, G.A., (1970): "Some Recent Analyses of Plume Rise Observations", International Air Poll. Confer., Wash., D.C., Dec.1970.

Briggs, G.A., (1972): Discussion on Chimney Plumes in Neutral and Stable Surroundings, Atmospheric Environment Vol. 6: 507-510.

Briggs, G.A., (1973): "Diffusion Estimation for Small Emissions," Air Resources Atmospheric Turbulence and Diffusion Laboratory, NOAA, Oak Ridge Contribution File No. 79 (Draft).

Briggs, G.A., (1975): "Plume Rise Predictions," Chapter 3 of Lectures on Air Pollution and Environmental Impact Analyses, D. A. Haugen, coordinator, American Meteorological Society.

Briggs, G.A., (1976): "Estimation of Downwash Effects," Chapter 6 of Power Generation: Air Pollution Monitoring and Control, Kenneth E. Noll and Wayne T. Davis, ed., Ann Arbor Science Publishers, Inc.

Briggs, G.A., (1981): Lecture on "Plume Rise" at the University of Illinois-Center for Environmental Study, November 24, 1981.

Briggs, G.A., (1982): "Plume Rise and Buoyancy Effects," Chapter 8 in Atmospheric Science and Power Production (Draft), ATDL Contribution File No. 79/6.

Fay, J.A., M. Escudier, and D.P. Holt, (1970): "A Correlation of Field Observations of Plume Rise", Journal of the Air Pollution Control Association, Vol. 20, No. 6: 391.

Gifford, F.A., Jr., (1972): "Atmospheric Transport and Dispersion over Cities", Nuclear Safety, Vol. 13: 395.

Halitsky, J., (1978): "Comment on Stack Downwash Prediction Formula," Atmospheric Environment Vol. 12: 1575.

Huber, A.H., et al., (1980): The Effects of a Squat Building on Short Stack Effluents: A Wind Tunnel Study, EPA-600/4-80-055.

Overcamp, T.J., and D.P. Hoult, (1971): "Precipitation in the Wake of Cooling Towers", Atmospheric Environment, Vol. 5: 751.

Pierce, T.E., and D.B. Turner, (1980): User's Guide for MPTER - A Multiple Point Gaussian Dispersion Algorithm with Optional Terrain Adjustment, EPA-600/8-80-016.

References

Sherlock, R.H., and F.A. Stalker, (1941): "A Study of Flow Phenomena in the Wake of Smokestacks," Dept. of Engineering Research, U. of Mich., Eng. Res. Bull. No. 29.

Strom, G.H., (1976): "Transport and Diffusion of Stack Effluents," Chapter 9 of Air Pollution, Volume I - Air Pollutants, their Transformation and Transport, Arthur C. Stern, ed., Academic Press, Inc.

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Appendix A
Computer Subroutine - Plume Rise From Small Point Sources

DDA:sc4381c,sp

```
0001          SUBROUTINE BEH072 (HF,HX,HMW,F,DELHF,DISTF,DELHX,HP,TS,VS,D,VF,  00001800
             1 KST,U,X,DIH07,T,P,701)                                          00001810
C       BEH072  (BRIGGS EFFECTIVE HEIGHT)  OCTOBER 1972                        00001820
C       D. B. TURNER, RESEARCH METEOROLOGIST* MODEL APPLICATIONS BRANCH        00001830
C        METEOROLOGY LABORATORY, ENVIRONMENTAL PROTECTION AGENCY.              00001840
C          ROOM 3168, NOAA BUILDING, RTP, PHONE (919) 549-8411 EXT 45600001850
C       MAILING ADDRESS  MTL,EPA, RESEARCH TRIANGLE PARK, NC 27711.           00001860
C        * ON ASSIGNMENT FROM NATIONAL OCEANIC AND ATMOSPHERIC                 00001870
C             ADMINISTRATION, DEPARTMENT OF COMMERCE.                         00001880
C       THIS DIFFERS FROM THE AUGUST 1972 VERSION IN STATEMENT 24 + 1 00001890
C        THE CONSTANT 2.4 PREVIOUSLY WAS 2.9, AND IN STATEMENT 27              00001900
C        THE CONSTANT 3.14159 PREVIOUSLY WAS 2.4 .                             00001910
C       THIS VERSION OF BRIGGS EFFECTIVE HEIGHT TO CALCULATE PLUME RISE00001920
C        FROM A SINGLE SOURCE IS BASED ON                                      00001930
C          1) BRIGGS,GARY A., 1971  SOME RECENT ANALYSES OF PLUME RISE 00001940
C               OBSERVATION. PP 1029 - 1032 IN PROCEEDINGS OF THE SECO00001950
C               INTERNATIONAL CLEAN AIR CONGRESS, EDITED BY H. M. ENGLUN00001960
C               AND A. T. BEERY. ACADEMIC PRESS, NEW YORK.                    00001970
C          2) BRIGGS, GARY A.,1972  DISCUSSION ON CHIMNEY PLUMES IN    00001980
C               NEUTRAL AND STABLE SURROUNDINGS. ATMOS. ENVIRON. 6, 507 00001990
C               - 510. (JUL 72).                                              00002000
C....
C       THE ORIGINAL BEH072 HAS BEEN MODIFIED BY IEPA IN 7/82 TO
C        ENABLE THE USER TO ACCOUNT FOR STACK TIP DOWNWASH AND
C        MECHANICALLY INDUCED TURBULENCE.  THE PROGRAM NOW REQUIRES
C        THE INPUT OF Z0 (SURFACE ROUGHNESS).  CALCULATIONS THAT HAVE
C        BEEN ADDED TO THE SUBROUTINE ARE BRACKETED BY "C...."
C        STATEMENTS AND DOCUMENTED.  THE PROGRAM IS CONSISTENT WITH
C        THE 7/82 PAPER BY D'AUREN/LAWLER WHICH INCORPORATES THE WORK
C        OF BRIGGS '75, BRIGGS '81, OVERCAMP AND MOULT '72.  ONLY
C        FINAL PLUME RISE (NOT TRANSITIONAL RISE) IS CALCULATED WHEN
C        STACK TIP DOWNWASH OR MECHANICALLY INDUCED TURBULENCE OCCUR.
C....
C
C        OUTPUT VARIABLES ARE...                                               00002010
C          HF    FINAL EFFECTIVE PLUME HEIGHT (METERS)                        00002020
C          HX    EFFECTIVE PLUME HEIGHT FOR DISTANCE X (METERS)               00002030
C          HMW   HEAT OUTPUT OF SOURCE (MW)                                   00002040
C          F     BUOYANCY FLUX (M**4/SEC**3)                                  00002050
C          DELHF FINAL PLUME RISE (METERS)                                    00002060
C          DISTF DISTANCE OF FINAL PLUME RISE FROM SOURCE (KM)                00002070
C          DELHX PLUME RISE AT DISTANCE X (METERS)                            00002080
C        INPUT VARIABLES ARE...                                               00002090
C          HP    PHYSICAL STACK HEIGHT (METERS)                               00002100
C          TS    STACK GAS TEMPERATURE (DEG K)                                00002110
C          FPN   FROUDE NUMBER
C          R     VELOCITY RATIO
C          VS    STACK GAS EXIT VELOCITY (M/SEC)                              00002120
C          D     INSIDE STACK DIAMETER (METERS)                               00002130
C          VF    STACK GAS VOLUMETRIC FLOW RATE (M**3/SEC)                    00002140
C          VK    VON KARMAN CONSTANT (0.4)
C          UST   FRICTION VELOCITY (M/SEC)
C          KST   STABILITY (CLASS), SEE PAGE 209 OF PASQUILL,                 00002150
C                 ATMOSPHERIC DISPERSION.  CLASSES DEFINED BY...              00002160
C                   1 TS PASQUILL STABILITY CLASS A                           00002170
C                   2 TS PASQUILL STABILITY CLASS B                           00002180
C                   3 TS PASQUILL STABILITY CLASS C                           00002190
C                   4 TS PASQUILL STABILITY CLASS D                           00002200
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                C                   S IS PASQUILL STABILITY CLASS E                                    00002210
                C                   S IS PASQUILL STABILITY CLASS F                                    00002220
                C          RCRIT1   VALUE ABOVE WHICH NO DOWNWASH OCCURS
                C          RCRIT2   VALUE BELOW WHICH STRONG DOWNWASH OCCURS
                C          U        WIND SPEED (M/SEC)                                                00002230
                C          X        DOWNWIND DISTANCE (KM)                                            00002240
                C          Z0       SURFACE ROUGHNESS (M)
                C          DTHDZ    POTENTIAL TEMPERATURE LAPSE RATE (DEG K/METER)                    00002250
                C          T        AMBIENT AIR TEMPERATURE (DEG K)                                   00002260
                C          P        AMBIENT AIR PRESSURE (MB)                                         00002270
                C
                C        THANKS TO DALE COVENTRY FOR HIS HELPFUL DISCUSSION ON                       00002280
                C         PROGRAMMING PLUME RISE, TO ROGER THOMPSON FOR THE COMMENT                  00002290
                C         CARDS, AND TO RUSS LEE WHO REVISED THIS ACCORDING TO REFERENCE             00002300
                C....
                C....
                C        VK IS THE VON KARMAN CONSTANT DEFINED AS 0.4
      0002            VK = 0.4
                C....
                C
      0003            IF(T)1,1,2                                                                     00002310
                C        T = 0. MEANS NO AMBIENT TEMPERATURE GIVEN.  USE T = 293.                    00002320
      0004        1 T = 293.                                                                         00002330
      0005        2 IF(P)3,3,4                                                                       00002340
                C        P = 0. MEANS NO AMBIENT AIR PRESSURE GIVEN.  USE P = 960.                   00002350
      0006        3 P = 960.                                                                         00002360
                C        IF VF IS NOT GIVEN, CALCULATE IT FROM STACK DATA.                           00002370
      0007        4 IF(VF)5,5,6                                                                      00002380
      0008        5 VF = 0.785398*VS*D*D                                                             00002390
                C        THE CONSTANT 0.785398 = PI/4                                                00002400
      0009        6 F = 3.12139*VF*(TS-T)/TS                                                         00002410
      0010          HPP = HP
                C....
                C....
                C        CALCULATION OF PARAMETERS FOR DOWNWASH
                C          THE FOLLOWING LINES TO "C...." HAVE BEEN ADDED
                C
      0011          IF (D.LE.0.) D = 0.1
                C       CALCULATE FROUDE NUMBER
      0012          FRN = VS/(9.80616*D*(TS-T)/TS)**0.5
                C       CALCULATE THE VELOCITY RATIO
      0013          R = VS/U
                C       CALCULATE VALUES OF R(CRITICAL)
      0014          RCRIT1 = 0.69 * FRN ** 0.47
      0015          RCRIT2 = 0.50 * FRN ** 0.65
                C....
                C....
                C        THE CONSTANT 3.12139 IS THE ACCELERATION DUE TO GRAVITY / PI.              00002420
      0016          HMN = 0.00011217*F*P                                                           00002430
                C        THE CONSTANT 0.00011217 = PI TIMES THE SPECIFIC HEAT OF AIR AT             00002440
                C         CONSTANT PRESSURE (0.24 CAL/GM*DEG K) TIMES MOLECULAR WEIGHT              00002450
                C         OF AIR (28.966 GM/GM.MOLE) DIVIDED BY IDEAL GAS CONSTANT                  00002460
                C         (0.0831 MB*M**3/GM.MOLE*DEG K) AND ACCELERATION DUE TO GRAVITY            00002470
                C         (9.80616 M/SEC*SEC) AND THEN MULTIPLIED BY (4.1855E-06 MW/CAL             00002480
                C         PER SEC) TO CONVERT THE ANSWER TO MEGAWATTS.                              00002490
                C        GO TO APPROPRIATE BRANCH FOR STABILITY CONDITION GIVEN.                    00002500
                C        IF UNSTABLE OR NEUTRAL GO TO 150, IF STABLE GO TO 20.                      00002510
      0017          GO TO (150,150,150,150,20,20,20),KST                                           00002520
```

```
                C       DETERMINE APPROPRIATE FORMULA FOR CALCULATING XST, DISTANCE AT 00002530
                C         WHICH TURBULENCE BEGINS TO DOMINATE.  THE FORMULA USED DEPENDS00002540
                C         UPON BUOYANCY FLUX. STATEMENTS 8 AND 9 ARE EQUATION (7).      00002550
                C....
                C....
                C       POTENTIAL DOWNWASH CALCULATIONS
                C         THE FOLLOWING LINES TO "C...." HAVE BEEN ADDED
                C
0018            150   IF (R - 1.5) 100,210,210
0019            210   CONTINUE
0020                  HPS = HP
0021                  IF (Z0.LE.0) Z0 = 0.1
0022                  IF(HP.LE.Z0) HP = 10.
                C       CALCULATE THE FRICTION VELOCITY
0023                  UST = (VK * U) / (ALOG(HP/Z0))
0024                  HP = HPS
0025            201   DELHF1 = 1.54 * (F/(U*UST**2.0)) ** 0.66667 * HP ** 0.3333
0026                  GO TO 7
0027            100   IF (R.GE.1.0.AND.R.GE.RCRIT1) GO TO 210
0028            101   IF (R.GE.1.0.AND.R.LT.RCRIT1) GO TO 102
0029                  GO TO 103
0030            102   HPRIME = 2.0 * (R - 1.5) * D
0031                  HP = HP + HPRIME
0032                  GO TO 210
0033            103   IF (R.LT.1.0.AND.R.GE.RCRIT1) GO TO 210
0034                  IF (R.LT.RCRIT1.AND.R.GE.RCRIT2) GO TO 102
0035                  HPRIME = 2.0 * (R - 1.5) * D
0036                  HP = HP + HPRIME
0037                  DELHF = 0.0
0038                  GO TO 31
                C
                C       CONTINUE STANDARD BRIGGS PLUME RISE CALCULATIONS
                C....
                C....
0039            7     IF(F-55.)8,8,9
0040            8     XST=14.*F**.625                                          00002560
0041                  GO TO 10                                                 00002570
0042            9     XST=34.*F**.4                                            00002580
0043            10    DISTF=3.5*XST                                            00002590
0044                  DELHF=1.6*F**0.333333*DISTF**0.666667/U                  00002600
0045                  IF(X)29,29,32                                            00002610
                C       IF X = 0.0, CALCULATE FINAL RISE ONLY, IF X IS GREATER THAN 00002620
                C         0.0, CALCULATE RISE FOR DISTANCE = X ALSO.          00002630
0046            32  XM = 1000.* X                                            00002640
                C       XM IS X IN METERS.                                     00002650
                C       STATEMENT 14 IS EQUATION (6), REFERENCE 1.             00002660
0047            14  DELHX = 1.6*F**0.333333*XM**0.666667/U                    00002670
0048                IF(DELHX.GT.DELHF)DELHX=DELHF                            00002680
                C                                                              00002690
                C....
                C       DETERMINE LESSER VALUE OF DELHF1 AND DELHF
0049                IF (DELHF1.LT.DELHF) DELHF = DELHF1
                C....
                C
0050                GO TO 30
0051            29  IF(DTHDZ)21,21,24                                        00002700
                C       IF DTHDZ IS NEGATIVE OR ZERO ASSIGN TO IT A VALUE OF 0.02 OR 00002710
                C         0.035 IF STABILITY IS SLIGHTLY STABLE OR STABLE, RESPECTIVELY.00002720
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0052                21 GO TO (7,7,7,7,22,23,231),KST                                  00002740
0053                22 DTHDZ = 0.02                                                   00002750
0054                   GO TO 24                                                       00002760
0055                23 DTHDZ = 0.035                                                  00002770
0056                24 S = 9.8066*DTHDZ/T                                             00002780
                C         THE CONSTANT 9.8066 IS THE ACCELERATION DUE TO GRAVITY.    00002790
                C         S IS A STABILITY PARAMETER.                               00002800
                C         CALCULATE PLUME RISE ACCORDING TO EQUATION (4), REFERENCE 1. 00002810
0057               DHA = 2.4*(F/(U*S))**0.333333                                     00002820
                C         CALCULATE PLUME RISE BY EQUATION (5), REFERENCE 1 FOR LIGHT 00002830
                C          AND CONDITIONS ACCORDING TO MORTON, TAYLOR, AND TURNER.  00002840
0058               DELHF = 5.0*F**0.25/S**0.375                                      00002850
0059               IF(DHA-DELHF) 25,25,27                                            00002860
0060            25 DELHF = DHA                                                       00002870
                C         DISTANCE TO FINAL PLUME RISE IS GIVEN BY THE FOLLOWING    00002880
0061            27 DISTF = 3.14159*U/S**0.5                                          00002890
                C         IF X = 0.0, CALCULATE FINAL RISE ONLY, IF X IS GREATER THAN 00002900
                C          0.0, CALCULATE RISE FOR DISTANCE = X ALSO.               00002910
                C         IF X IS ZERO OR LESS, GO TO 29 AND SET PLUME RISE AND DIST. TO 00002920
                C          MAXIMUM PLUME RISE EQUAL TO ZERO.                       00002930
0062               IF(X)29,29,33                                                     00002940
0063            33 XM = 1000.*X                                                      00002950
                C         XM IS X IN METERS.                                        00002960
                C         IF XM IS GREATER THAN THE DISTANCE TO THE POINT OF FINAL PLUME 00002970
                C          RISE, SET PLUME RISE EQUAL TO FINAL PLUME RISE, OTHERWISE, 00002980
                C          CALCULATE PLUME RISE FROM EQUATION (6),REFERENCE 1.     00002990
0064               IF(XM-DISTF)14,14,28                                              00003000
0065            28 DELHX = DELHF                                                     00003010
0066               GO TO 30                                                          00003020
0067            29 DELHX = 0.                                                        00003030
0068               HX = 0.                                                           00003040
0069               GO TO 31                                                          00003050
                C         CALCULATE EFFECTIVE HEIGHT AT DISTANCE X.                 00003060
0070            30 HX = HP + DELHX                                                   00003070
                C         CALCULATE FINAL EFFECTIVE HEIGHT.                         00003080
0071            31 HF = HP + DELHF                                                   00003090
0072               DISTF = DISTF/1000.                                               00003100
0073               HP = HPP                                                          00003110
0074               RETURN                                                            00003120
0075               END
```